$$\mathcal{E}_{LOC} = \mathcal{E}^0_{LOC}(S) + \beta(E_{11} + E_{22} + E_{33}) + \frac{1}{2} C_{11}(E_{11}^2 + E_{22}^2 + E_{33}^2)$$

$$+ C_{12}(E_{11}E_{22} + E_{22}E_{33} + E_{33}E_{11}) + 2C_{44}(E_{23}^2 + E_{31}^2 + E_{12}^2)$$

$$+ \frac{1}{6} C_{111}(E_{11}^3 + E_{22}^3 + E_{33}^3) + \frac{1}{2} C_{112}(E_{11}^2(E_{22} + E_{33}) + E_{22}^2(E_{33} + E_{11})$$

$$+ E_{33}^2(E_{11} + E_{22})) + C_{123}E_{11}E_{22}E_{33} + 2C_{144}(E_{11}E_{23}^2 + E_{22}E_{31}^2 + E_{33}E_{12}^2)$$

$$+ 2C_{155}((E_{11} + E_{22})E_{12}^2 + (E_{22} + E_{33})E_{23}^2 + (E_{33} + E_{11})E_{31}^2$$

$$+ b_{11}(E_{11}\alpha_1^{*2} + E_{22}\alpha_2^{*2} + E_{33}\alpha_3^{*2}) + 2b_{44}(E_{23}\alpha_2^*\alpha_3^* + E_{31}\alpha_3^*\alpha_1^*$$

$$+ E_{12}\alpha_1^*\alpha_2^*) + \frac{1}{2} B_{111}(E_{11}^2\alpha_1^{*2} + E_{22}^2\alpha_2^{*2} + E_{33}^2\alpha_3^{*2}) + B_{123}(E_{11}E_{22}\alpha_3^{*2}$$

$$+ E_{22}E_{33}\alpha_1^{*2} + E_{33}E_{11}\alpha_2^{*2}) + 2B_{144}(E_{11}E_{23}\alpha_2^*\alpha_3^* + E_{22}E_{31}\alpha_3^*\alpha_1^*$$

$$+ E_{33}E_{12}\alpha_1^*\alpha_2^*) + 2B_{155}((E_{11} + E_{22})E_{12}\alpha_1^*\alpha_2^* + (E_{22} + E_{33})E_{23}\alpha_2^*\alpha_3^*$$

$$+ (E_{33} + E_{11})E_{31}\alpha_3^*\alpha_1^*) + 2B_{441}(E_{23}^2\alpha_1^{*2} + E_{31}^2\alpha_2^{*2} + E_{12}^2\alpha_3^{*2})$$

$$+ 4B_{456}(E_{23}E_{31}\alpha_1^*\alpha_2^* + E_{31}E_{12}\alpha_3^*\alpha_2^* + E_{12}E_{23}\alpha_1^*\alpha_3^*) + K_1(\alpha_1^{*2}\alpha_2^{*2}$$

$$+ \alpha_2^{*2}\alpha_3^{*2} + \alpha_3^{*2}\alpha_1^{*2}) + \frac{2\pi\lambda}{M_s^2}\left(\left|\vec{\nabla}_a\alpha_1\right|^2 + \left|\vec{\nabla}_a\alpha_2\right|^2 + \left|\vec{\nabla}_a\alpha_3\right|^2\right) \qquad (2.13)$$

Keeping terms to lowest order, although not entirely consistently, one obtains the original expression of Becker and Doring[3] from conventional magnetoelastic theory plus the exchange energy.

$$\mathcal{E}_{LOC} = \frac{1}{2} c_{11}(e_{11}^2 + e_{22}^2 + e_{33}^2) + c_{12}(e_{11}e_{22} + e_{22}e_{33} + e_{33}e_{11})$$
$$+ 2c_{44}(e_{23}^2 + e_{31}^2 + e_{12}^2) + b_1(e_{11}\alpha_1^2 + e_{22}\alpha_2^2 + e_{33}\alpha_3^2)$$
$$+ 2b_2(e_{12}\alpha_1\alpha_2 + e_{23}\alpha_2\alpha_3 + e_{31}\alpha_3\alpha_1) + K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2$$
$$+ \alpha_3^2\alpha_1^2) + A\left(\left|\vec{\nabla}\alpha_1\right|^2 + \left|\vec{\nabla}\alpha_2\right|^2 + \left|\vec{\nabla}\alpha_3\right|^2\right) \qquad (2.14)$$

where $b_1 = b_{11}$, $b_2 = b_{44}$, and $A = 2\pi\lambda/M_s^2$.

There is a reason for developing the energy expression through a finite strain formalism. The conventional magnetoelastic energy expression, Equation (2.14), was obtained by adding the energy of a magnetic rigid solid to the energy of a nonmagnetic elastic solid and then superposing an interaction term to describe the magnetoelastic effect. It has been pointed out that this expression does not contain sufficient terms to properly account for the energy to the order of strain assumed.[5] The success of the conventional magnetoelastic expression can be attributed to the extremely low strains ($\sim 10^{-5}$) existing in usual magnetostrictive phenomenon. One worries whether it will be sufficient to describe the behavior for the quite high strains ($\sim 10^{-2}$) which prevail in the present inverse magnetostrictive effect. Although conventional magnetoelastic theory will be used in the subsequent chapter, the effect of finite strain will be seriously considered in the appendix.

In summary, a complete energy expression for an anisotropic ferromagnet has been obtained.

$$E = \int (\mathcal{E}_{LOC} - \frac{1}{2}\vec{H}_d \cdot \vec{M} - \vec{H}_e \cdot \vec{M})\, dV \qquad (2.15)$$

With this expression and thermodynamic equilibrium and stability criteria, magnetic equilibrium properties can, in principle, be predicted.